

Quantization of electric charge, the neutrino and generation nonuniversality

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(February 1, 2008)

It is showed that the electric charge quantization is unconnected to Majorana neutrino in the non-universal generations leptoquark-bilepton flavordynamics which includes the right-handed neutrino and an explicit U(1) factor in the gauge semisimple group.

Two fundamental questions leaved open within the standard model of nuclear and electromagnetic interactions [1] are the flavor question and the fermion mass hierarchy, being addressed [2–6] in the leptoquark–bilepton flavordynamics [2,3,7] having the local gauge symmetries $G_{3m1} \equiv \text{SU}(3)_c \otimes \text{SU}(m)_L \otimes \text{U}(1)_{L+R}$ with $m = 3, 4$. There is also a cubic seesaw relation $m(\nu_\ell) = m^3(\ell)/M_W^2$ constraining the neutrino mass to the charged lepton masses [8] implicating the 10^{-5} eV value for the lightest neutrino mass, and also the interesting possibility of double beta decay depending less on the neutrino mass [9] than in many extensions of the standard model, as well as the associated Majoron emission process [10]. It was also showed the non-trivial issue that in the $m = 3$ model there is a Peccei–Quinn symmetry [11] with an invisible axion, solving the strong-CP problem [12]. Although in the minimal G_{331} leptoquark-bilepton model the massive neutrinos are Majorana fermions [13,14], due to the non-universal generations structure the electric charge quantization and Majorana neutrino connection is lost. Such fact agrees with the pioneering scrutiny [15] involving a large class of gauge models containing a U(1) factor in the gauge group, the right-handed neutrino and generations universality. Using the lightest leptons as the particles which determine the approximate symmetry with the generations nonuniversality and if lepton charges are only 0, ± 1 , SU(4) is the highest symmetry group to be included in the electroweak sector. There is no room for $m > 4$. A model with the SU(4) \otimes U(1) symmetry was proposed more than one decade ago by Voloshin [16] but quarks were not included there.

Let us consider the largest leptoquark–bilepton gauge semisimple G_{341} group extension. The electric charge operatorial formula [17]

$$\frac{Q}{|e|} = \Lambda_3 + \xi \Lambda_8 + \zeta \Lambda_{15} + \varsigma N \Lambda_0 \quad (1)$$

is embedded in the traceless neutral generators Λ_i , $i = 3, 8, 15$ of the SU(4) gauge group,

$$\Lambda_3 = \frac{1}{2} \text{diag}(1, -1, 0, 0), \quad (2a)$$

$$\Lambda_8 = \frac{1}{2\sqrt{3}} \text{diag}(1, 1, -2, 0), \quad (2b)$$

$$\Lambda_{15} = \frac{1}{2\sqrt{6}} \text{diag}(1, 1, 1, -3) \quad (2c)$$

and $\Lambda_0 = \text{diag}(1, 1, 1, 1)$, with the specific embedding parameters $\xi = -1/\sqrt{3}$, $\zeta = -2\sqrt{6}/3$, $\varsigma = 1$, where N is the new charge associated to the symmetric non-chiral Abelian semisimple factor of the G_{341} local symmetry. In the $\bar{4}$ representation the neutral generators are [18]

$$\bar{\Lambda}_3 = \frac{1}{2} \text{diag}(0, 0, 1, -1), \quad (3a)$$

$$\bar{\Lambda}_8 = \frac{1}{2\sqrt{3}} \text{diag}(0, 2, -1, -1), \quad (3b)$$

$$\bar{\Lambda}_{15} = \frac{1}{2\sqrt{6}} \text{diag}(3, -1, -1, -1). \quad (3c)$$

The SU(4) maximal subalgebras are SU(3) \otimes U(1), SU(2) \otimes SU(2) \otimes U(1), Sp(2), and SU(2) \otimes SU(2). The isomorphism is SU(4) \sim SO(6). Three families of independent leptonic chiral flavor gauge symmetry eigenstates tranforms collectively,

$$f_{\ell L} = \left(\begin{pmatrix} \nu_{\ell L} \\ \ell_L \\ \nu_{\ell R} \\ \ell_R \end{pmatrix}^c \right) \sim (\mathbf{1}_c, \mathbf{4}_L, N = 0) \quad (4)$$

with the label $\ell = e, \mu, \tau$ and the charge conjugated fields $\ell^c = C \bar{\ell}^T$, ν_ℓ^c , being $C \equiv i\gamma^2 \gamma^0$. One quark family has the attributions

$$Q_{1L} = \left(\begin{pmatrix} u_1 \\ d_1 \\ u' \\ J \end{pmatrix} \right)_L \sim (\mathbf{3}_c, \mathbf{4}_L, +2/3) \quad (5)$$

with the singlets

$$u_{1R} \sim (\mathbf{3}_c, \mathbf{1}_R, +2/3), \quad (6a)$$

$$d_{1R} \sim (\mathbf{3}_c, \mathbf{1}_R, -1/3), \quad (6b)$$

$$u'_{1R} \sim (\mathbf{3}_c, \mathbf{1}_R, +2/3), \quad (6c)$$

$$J_R \sim (\mathbf{3}_c, \mathbf{1}_R, +5/3), \quad (6d)$$

and the other two families have the following common transformation properties

$$Q_{\alpha L} = \begin{pmatrix} j_\alpha \\ d'_\alpha \\ u_\alpha \\ d_\alpha \end{pmatrix}_L \sim (\mathbf{3}_c, \bar{\mathbf{4}}_L, -1/3), \quad (7)$$

$\alpha = 2, 3$ for the chiral left-handed fields and

$$j_{\alpha R} \sim (\mathbf{3}_c, \mathbf{1}_R, -4/3), \quad (8a)$$

$$d'_{\alpha R} \sim (\mathbf{3}_c, \mathbf{1}_R, -1/3), \quad (8b)$$

$$u_{\alpha R} \sim (\mathbf{3}_c, \mathbf{1}_R, +2/3), \quad (8c)$$

$$d_{\alpha R} \sim (\mathbf{3}_c, \mathbf{1}_R, -1/3), \quad (8d)$$

where u' , J , j_α and d'_α are new quark flavors with electric charges $+\frac{2}{3}|e|$, $+\frac{2}{3}|e|$, $-\frac{4}{3}|e|$ and $-\frac{1}{3}|e|$ respectively, where the down-like quark flavors transport the quantum of electric charge being

$$\begin{aligned} |e| &= \frac{gt}{(1+4t^2)^{1/2}} = \frac{g'}{(1+4t^2)^{1/2}} \\ &= 1.602\,176\,462(63) \times 10^{-19} \text{ C} \\ &= 4.803\,204\,20(19) \times 10^{-10} \text{ esu} \end{aligned} \quad (9)$$

the proton charge [20], and $t \equiv g'/g$, where g and g' are the SU(4) and U(1) gauge coupling constants. The electric charged and neutral leptons acquire mass through the symmetric decuplet $(\mathbf{1}_c, \bar{\mathbf{10}}_S, 0)$ of scalar fields,

$$\bar{H} = \begin{pmatrix} H_1^0 & H_1^+ & H_2^0 & H_2^- \\ H_1^+ & H_1^{++} & H_3^+ & H_3^0 \\ H_2^0 & H_3^+ & H_4^0 & H_4^- \\ H_2^- & H_3^0 & H_4^- & H_2^{--} \end{pmatrix} \quad (10)$$

in the Yukawa interactions

$$\mathcal{L}_Y^f = -\frac{1}{2} G_{\ell\ell'} \overline{(f_{\ell L})^c} \bar{H} f_{\ell' L} \quad (11)$$

having the general form of a Majorana mass term after the spontaneous symmetry breaking. Since the lepton mass term transforms as $(f_{\ell L})^c f_{\ell' R} \sim (\mathbf{1}_c, \mathbf{4} \otimes \mathbf{4}, 0) = (\mathbf{1}_c, \mathbf{6}_A \oplus \mathbf{10}_S, 0)$ and the sextet will leave one lepton massless and two others degenerate for three families, it is necessary to introduce the $N = 0$ symmetric decuplet which plays no role in generating quark masses. The explicit fermion bilinears and Higgs bosons Yukawa couplings are

$$\begin{aligned} \overline{(f_L)^c} \bar{H} f_L = & \overline{(\nu_\ell^c)}_R \nu_{\ell R} H_1^0 + \overline{(\ell^c)}_R \nu_{\ell L} H_1^+ + \bar{\nu}_{\ell R} \nu_{\ell L} H_2^0 + \bar{\ell}_R \nu_{\ell L} H_2^- \\ & + \overline{(\nu_\ell^c)}_R \ell_L H_1^+ + \overline{(\ell^c)}_R \ell_L H_1^{++} + \bar{\nu}_{\ell R} \ell_L H_3^+ + \bar{\ell}_R \ell_L H_3^0 \\ & + \overline{(\nu_\ell^c)}_R \nu_{\ell L} H_2^0 + \overline{(\ell^c)}_R \nu_{\ell L} H_3^+ + \bar{\nu}_{\ell R} \nu_{\ell L} H_4^0 + \bar{\ell}_R \nu_{\ell L} H_4^- \\ & + \overline{(\nu_\ell^c)}_R \ell_L^c H_2^- + \overline{(\ell^c)}_R \ell_L^c H_3^0 + \bar{\nu}_{\ell R} \ell_L^c H_4^- + \bar{\ell}_R \ell_L^c H_2^{--} \end{aligned} \quad (12)$$

and after the spontaneous symmetry breaking steps

$$G_{341} \rightarrow G_{331} \rightarrow G_{321} \rightarrow G_{31} \quad (13)$$

remain the following mass terms,

$$\begin{aligned} -\mathcal{L}_Y^{\text{mass}} = & \frac{1}{2} (G_{\nu_\ell \nu_{\ell'}} \langle H_1^0 \rangle_0 \overline{(\nu_\ell^c)}_R \nu_{\ell' L} + G_{\nu_\ell \nu_{\ell'}} \langle H_2^0 \rangle_0 \bar{\nu}_{\ell R} \nu_{\ell' L} \\ & + G_{\ell \ell'} \langle H_3^0 \rangle_0 \bar{\ell}_R \ell'_L + G_{\nu_\ell^c \nu_{\ell'}^c} \langle H_2^0 \rangle_0 \overline{(\nu_\ell^c)}_R \nu_{\ell' L}^c \\ & + G_{\nu_\ell \nu_{\ell'}} \langle H_1^0 \rangle_0 \bar{\nu}_{\ell R} \nu_{\ell' L}^c + G_{\ell \ell'} \langle H_3^0 \rangle_0 \overline{(\ell^c)}_R \ell'_L \end{aligned} \quad (14)$$

and the neutrinos are Dirac–Majorana particles [21].

The vanishing anomalies conditions containing the $U(1)_N$ fermionic attributions imply the following constraints between the N 's:

$$\begin{aligned} \text{Tr}([\text{SU}(3)_c]^2 [\text{U}(1)_N]) &= 0 : \\ 3(N_{Q_{1L}} + 2N_{Q_{\alpha L}}) - 3(N_{U_{1,\alpha R}} + N_{D_{1,\alpha R}}) \\ &- N_{J_R} - 2N_{j_{\alpha R}} - N_{u'_R} - 2N_{d'_{\alpha R}} = 0, \end{aligned} \quad (15a)$$

$$\begin{aligned} \text{Tr}([\text{SU}(4)_L]^2 [\text{U}(1)_N]) &= 0 : \\ 3(N_{Q_{1L}} + 2N_{Q_{\alpha L}}) + \sum_\ell N_\ell^3 &= 0, \end{aligned} \quad (15b)$$

$$\begin{aligned} \text{Tr}([\text{U}(1)_N]^3) &= 0 : \\ 3(N_{Q_{1L}}^3 + 2N_{Q_{\alpha L}}^3) - 3(N_{U_{1,\alpha R}}^3 + N_{D_{1,\alpha R}}^3) + \\ N_{J_R}^3 - 2N_{j_{\alpha R}}^3 - N_{u'_R}^3 - 2N_{d'_{\alpha R}}^3 + \sum_\ell N_\ell^3 &= 0, \end{aligned} \quad (15c)$$

$$\begin{aligned} \text{Tr}([\text{gravitational anomaly term}]^2 [\text{U}(1)_N]) &= 0 : \\ 3(N_{Q_{1L}} + 2N_{Q_{\alpha L}}) - 3(N_{U_{1,\alpha}} + N_{D_{1,\alpha}}) + \\ N_{J_R} - 2N_{j_{\alpha R}} - N_{u'_R} - 2N_{d'_{\alpha R}} + \sum_\ell N_\ell &= 0 \end{aligned} \quad (15d)$$

where $N_{U_{1,\alpha}}$ and $N_{D_{1,\alpha}}$ are the $U(1)_N$ quantum numbers of the standard quark flavors. The Witten global anomaly [22] does not involve the $U(1)_N$ quantum numbers. The three leptonic classical constraints are

$$N_\ell = 0 \quad (16)$$

coming from the Abelian gauge invariance of \mathcal{L}_Y where the electric charge quantization of the leptonic sector is already contained. From the quantum and classical gauge invariance constraints the electric charges of fundamental leptons and quarks, in the $|e|$ unit, are [23]

$$\begin{aligned} \mathcal{Q}_{\nu_\ell} &= 0, \quad \mathcal{Q}_\ell = -1, \quad \mathcal{Q}_D = -\frac{1}{3}, \\ \mathcal{Q}_U &= -2\mathcal{Q}_D = -2\mathcal{Q}_{d'_\alpha} = \mathcal{Q}_{u_1}, \\ \mathcal{Q}_J &= -5\mathcal{Q}_D, \quad \mathcal{Q}_{j_\alpha} = 4\mathcal{Q}_D \end{aligned} \quad (17)$$

even for massless neutrinos. There are no new constraints coming from the cancellation of mixed gauge and gravitational anomalies. If we consider the global symmetry associated to the conservation of the leptobaryon quantum number

$$\mathcal{F} = L + B = \sum_\ell L_\ell + B, \quad (18)$$

which prevents neutrinos from getting a mass to be explicitly broken then Majorana mass terms arise if

$\langle H_{1,4}^0 \rangle \neq 0$ turning-off the VEVs of the H_2^0 and H_3^0 non-diagonal fields. However, turning-on the VEVs of all neutral scalar fields the neutrinos are Dirac–Majorana fermions [21].

Now let us add right-handed neutrinos as gauge flavor singlets. If $\langle H_1^0 \rangle_0 = \langle H_2^0 \rangle_0 = \langle H_4^0 \rangle_0 = 0$, but $\langle H_3^0 \rangle_0 \neq 0$ for the charged lepton masses, the Dirac mass terms for the neutral fermions

$$-\mathcal{L}_Y^{\nu_\ell, \langle \eta \rangle_0} = G_{f_\ell \nu_\ell} \bar{f}_{\ell L} \langle \eta \rangle_0 \nu_{\ell' R} + \text{H.c.} \quad (19)$$

arise in the Yukawa couplings through the multiplet of scalar fields

$$\eta = \begin{pmatrix} \eta_1^0 \\ \eta_0^- \\ \eta_2^- \\ \eta_2^+ \end{pmatrix} \sim (\mathbf{1}_c, \mathbf{4}, 0) \quad (20)$$

also necessary in the quark sector,

$$-\mathcal{L}_Y^{Q, \eta} = F_{1k} \bar{Q}_{1L} U_{kR} \eta + F'_{\alpha k} \bar{Q}_{\alpha L} D_{kR} \bar{\eta} + \text{H.c.}, \quad k = 1, 2, 3 \quad (21)$$

where

$$\bar{\eta} = \begin{pmatrix} \eta_2^+ \\ \eta_2^- \\ \eta_0^- \\ \eta_1^0 \end{pmatrix} \sim (\mathbf{1}_c, \bar{\mathbf{4}}, 0). \quad (22)$$

The classical gauge invariance of the $\mathcal{L}_Y^{\nu_\ell, \langle \eta \rangle_0}$ leptonic terms implies $N_{\nu_\ell} = 0$. The L and B attributions of the leptoquark fermions are $L_{j\alpha} = -L_J = +2$, $B_J = B_{j\alpha} = +\frac{1}{3}$, and the bilepton gauge bosons have lepton number $L = \pm 2$. The bileptons [26] are contained also in the stable-proton SU(15) grand unified theory [27] with non-chiral fermions and anomaly cancellation through mirror fermions.

The quantization of electric charge is inevitable in the G_{3m1} models of leptoquark fermions and bilepton bosons [25,24] with three non-repetitive fermion generations breaking generation universality and does not depend on the character of the neutral fermions. Each generation is anomalous and are not replicas of one another so that the quantum anomalies cancel when the number of generations is a multiple of the number of color charges. There is not any connection between electric charge quantization and the massless Weyl, and the massive Dirac, Majorana or Dirac–Majorana fundamental neutral fermions. If the \mathcal{F} -symmetry is explicitly broken the neutrinos are Majorana fermions when $\langle H_{1,4}^0 \rangle_0 \neq 0$ and Dirac–Majorana fermions when all neutral components of the symmetric decuplet acquire their vacuum expectation values. This is another clue about the promising perspectives that neutrino address in the direction of new physics. The G_{3m1} leptoquark-bilepton models have the leptonic representation content structure of grand unified theories in which the electric charge operator contains only the $SU(m)_L$ diagonal generators whose number is the rank of the group.

ACKNOWLEDGMENTS

A.D. would like to thank the CAPES (Brazil) for full financial support.

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